

# Laboratory 7

## Measurement on Strain & Force

Department of Mechanical and Aerospace Engineering  
University of California, San Diego  
MAE170

Megan Ong  
Diana Wu Wong  
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**Abstract:**

The purpose of this experiment was to investigate the measurement of the strain and force using various different theoretical methods. The spring constants were calculated to be 467.79 N/m and 464.79 N/m respectively for the static method and the dynamic method. The Young Modulus was found to be  $42.7 \pm 0.4$  GPa and compared to the theoretical value 69 GPa, resulting a percent error of 38%. The moment of inertia was found to be  $4.45 \pm 0.04 \times 10^{-10}$ . The theoretical and experimental strains were found to have a similar correlation with a considerate discrepancy. The Gauge Factor for the experimental SG 3 was  $2.190 \pm 0.005$ , in comparison to the theoretical value was 2.08 with 5% error.

## Introduction:

The objective of this lab is to explore the properties of solid mechanics through a series of theories and equations. In this set up, a beam of aluminum is fixed in one end with strain gauges attached at different parts of the beam. These strain gauges will help create voltage outputs according to the extent of the strain and bent done by the loads. Strain gauges are etched from thin foil metal sheets that are bonded to plastic backing<sup>1</sup>. The purpose of the strain gauge is to detect resistance change at each portion of the beam at different for each load and eventually help determine the different strain and force properties of the beam. The strain gauge uses the Wheatstone Bridge to help accurately determine even small resistance changes. Therefore, it is convenient to use it on the measurement of the forces of the beam and calculate properties such as the spring constant, Poisson's ratio, Young's modulus and the gauge factor.

## Theory:

The first objective of the lab involves determining the beam deflection. When weight is applied to the free end, the force exerts a vertical deflection defined by the following equation, where E is the Young's modulus, F is the force applied at the free end, I is the moment of inertial described by equation (2), L is the longitude of the beam and x is the placement where it is calculated:

$$(1) y = \frac{F}{EI} \left( \frac{L^3}{3} - \frac{1}{2} L^2 x + \frac{1}{6} x^3 \right)$$
$$(2) I = \frac{bh^3}{12}$$

Conversely the maximum vertical deflection would be achieved at the free end when  $x=L$ , deriving to the next equation:

$$(3) y_{max} = \frac{FL^3}{3EI}$$

The Young's modulus, E, is derived from the force and displacement equation from equation 3 and the Hooke's Law of spring law at equation 5, where k is the spring constant:

$$(4) E = \frac{kL^3}{3I}$$
$$(5) F = ky$$

To find the spring constant k, two methods were used: static and dynamic method. The static method was already mentioned above at equation 5 using the Hooke's Law. In comparison the dynamic method uses the concept of oscillation to solve for k. The equation for oscillation angular frequency is the following, where  $\omega$  is the oscillation angular frequency  $\lambda L$  is 1.88 for first natural frequency, M is the beam mass and m is the load mass.

$$(6) \omega = (\lambda L)^2 \sqrt{\frac{k}{3(M+4.167m)}}$$

If the mass of the beam is neglected, then the oscillation of a mass can be found using the following equation:

$$(7) \omega = \sqrt{\frac{k}{m}}$$

To find the strain, Hooke's Law states that it is proportional to the stress, where the strain ( $\epsilon$ ) can be obtained in the change in length over the original length and E is the Young's Modulus or elastic modulus.

$$(8) \sigma = E\epsilon$$

For 3D solids, Poisson's ratio ( $\nu$ ) helps compare the transverse strain to the longitudinal strain with the following equation:

$$(9) \nu = \frac{\text{transverse strain}}{\text{longitudinal strain}} = \frac{\epsilon_t}{\epsilon_l}$$

All measurements are based on the results from the strain gauges. The principle follows that the length change of a wire results in a change of resistance<sup>1</sup>. R is the resistance,  $\rho$  is the resistivity of the material, L is the length and A is the area.

$$(11) A = \frac{\pi b^2}{4}; \text{ where } b \text{ is the diameter of the wire}$$

At last, using the principle of strain gauge along with the Poisson's ratio, the Gauge Factor is derived to the following, where  $\epsilon_l$  is the longitudinal strain,  $\Delta R$  is the change in resistance and R is the resistance:

$$(12) G.F = \frac{\frac{\Delta R}{R}}{\epsilon_l}$$

### Procedure:

First using the static method, the dimensions of the beam were recorded as well as the initial height of the free end of the beam from the bench top. Then the strain gauge was connected to the strain and Pressure and Conditioning Board. The board was powered with +15V, -15V, and ground from the Proto-Board. Additionally, the board contained a Wheatstone Bridge, to which the strain gauge was connected, completing the onboard bridge. The circuit was run for a few minutes and the voltage from the bridge was measured using a handheld DMM. This voltage was then nulled from future readings. To collect data for the strain voltage, ~200g were added to the free end of the beam. The resulting height difference and voltage output were then recorded. This process was repeated for 4 different weight measurements, and then done in reverse order for hysteresis analysis. When the process was completed for a single strain gauge, it was then repeated for the remaining gauges using the same weights.

Next in the dynamic method, the strain gauge reaction was displayed on the oscilloscope rather than the DMM. Using a load of ~400g, the strain gauge furthest from the fixed end was analyzed by gently pushing down on the beam causing it to oscillate. The oscillation was then traced onto the oscilloscope, from which the period was determined. This process was then repeated for 4 more weights.

### Result

The first part of the experiment involved measuring the beam deflection at each weight, then plot it against the bridge output voltage. The initial measurements of the set-up were as follows: The dimensions of the beam were 32.1mm width, 5.5 mm thickness and 497 mm long and the strain gauge were positioned as set in Table 1.

LSG1	LSG2	LSG3	LSG4	LSG5
305 mm	405 mm	415 mm	457 mm	457 mm

**Table 1:** Position of Strain Gauges.

Using the data collected in the static method, table 2, the load could be plotted against the change in height. The slope of the resulting Figure 1, 464.79 N/m, was the spring constant k, of the beam. This value could then be used in equation 4, to find Young's modulus, E along the moment of Inertia using equation 2 that was calculated to be  $4.45 \pm 0.04 \times 10^{-10}$ . For the data collected, E was found to be  $42.7 \pm 0.4$  GPa. The given value was 69GPa. On a single graph, all of the strain gauges were plotted bridge voltage against the load as seen in Figure 2 using all of the gauges in Table 2 appendix. Further discussion in regards to their relations will be addressed in the discussion section.

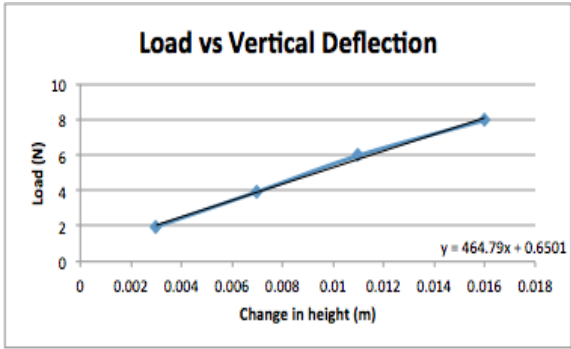


Figure 1: Load (N) against Vertical Deflection

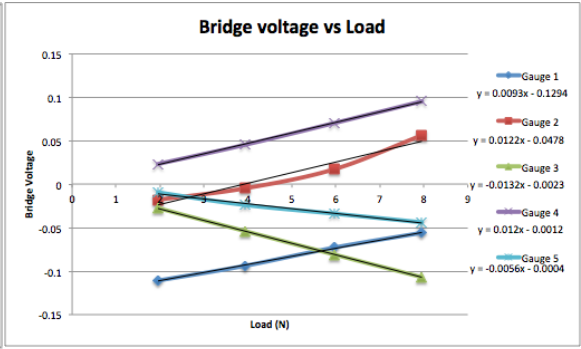


Figure 2: Bridge Voltage against all strain gauges

The experimental and theoretical strain for both SG1 and SG2 were then plotted as a function of load, as shown as Figure 3 and Figure 4 below. The relationship shows that both graphs have the same positive correlation. However, both of their experimental strains are vertically shifted down. SG1 shows a positive linear correlation for both instances, whereas SG2 shows concave up data points in comparison to its theoretical strain.

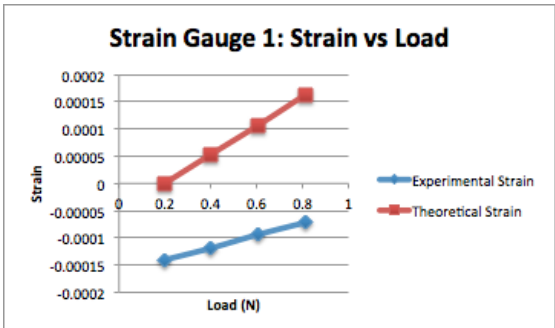


Figure 3: Experimental against theoretical Strain in relationship to the Load for SG 1.

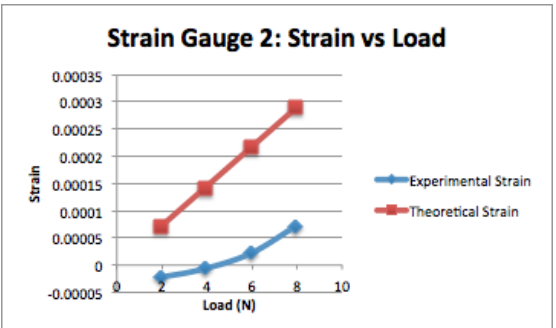


Figure 4: Experimental against theoretical Strain in relationship to the Load for SG 2.

Then the plot of the experimental strain between SG4 and SG5 was graphed in order to find the experimental Poisson's ratio. According to Figure 5, the measured Poisson's, the slope of the graph was 0.4716.

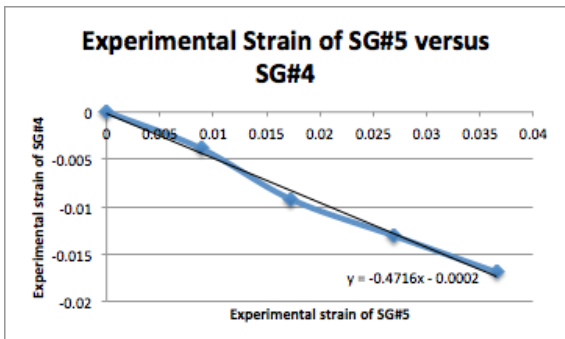
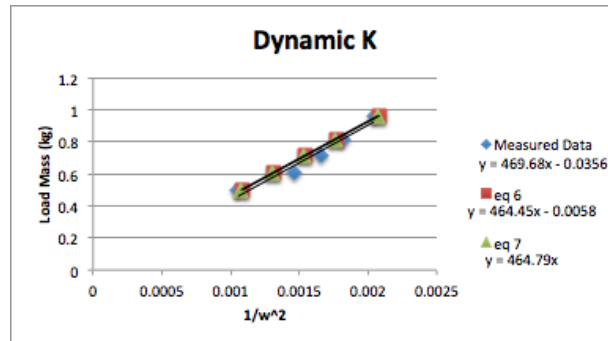


Figure 5: Experiment Strain of SG 4 and SG 6.

In the second part of the experiment, the objective was to use the dynamic method to find the spring constant  $k$  using the equation 6 of angular frequency. By graphing the load mass versus  $\frac{1}{\omega^2}$ , the slope of the line could be taken to find the spring constant. Graphing the measured data,  $k$  was found to be 469.68 N/m. However when the theoretical values were graphed against this, the

spring constant varied slightly. When the mass of the beam was taken into account, equation 6 was used and resulted in a spring constant of 464.45 N/m. When the mass of the beam was neglected, the spring constant was 464.79 N/m, found using equation 7. The graph of the oscillations for each load are shown in Appendix Figure 8-12.



**Figure 6:** Spring constant of the beam using equation 6 and equation 7

## Discussion

In the first part of the experiment, the resulting spring constant from Figure 1 of the load against change in height was 467.79 N/m. Then the value was inserted to equation 4 to find the Young's modulus  $42.7 \pm 0.4$  GPa. In comparison to the given value, 69 GPa, the percent error was 38.1%. This shows that the static method was an inaccurate approach to find the properties of the solids mechanics. The load on the beam should not exceed 1000 grams because of the limitations on the maximum elastic load from the strain gauges.

The bridge voltage,  $V_b$ , and the load of all 5 strain gauges were plotted against each other in Graph 2. All of the gauges follow a linear trend as is expected due to the resistance increasing as weight is added to the beam. Studying the trend lines of SG2 and SG3, the slopes of these 2 graphs are nearly the same but opposite in sign. This is to be expected as these gauges were placed at the same distance from the free end, however SG2 was placed at the top of the beam while SG3 was placed at the bottom of the beam. This will lead to a opposite trends because the tension in the beam due to the added weight would cause the resistance to increase, while at the same time the bottom of the beam would experience a compression, causing the resistance to decrease.

Then, the experimental and theoretical strain for both SG1 and SG2 were then plotted as a function of load, as shown as graph 3 and graph 4 in the results section. As mentioned before, both graphs show a positive correlation and shifted vertically down. The vertical shift shows a systematic experimental error, this is most likely an error from the set-up of the instrument. Regardless of the systematic error, the slope for both experimental and theoretical strain in both graph differ. Looking at SG 1 graph 3, although both data points had linear positive correlation, the slope of the data points for the experimental strain was smaller than the theoretical one. Similarly, the slope for the SG 2 graph 4, doesn't only show different in slope between the two data sets, but also the experimental strain showed to be concave up. The reason behind the discrepancy in the slope might be because the instrument is not as sensitive as expected from the given relation. Therefore the slopes for the experimental strains were lower than the theoretical one. The concave up phenomena for SG 2 might be due to the position of the SG 2 achieving its maximum elasticity, hence the larger the slope the larger the load was. As a result, SG 1 would be a more accurate measure for an inclinometer.

The calculated Gauge Factor for SG 3 was calculated by finding the optimal maximum strain and plugging it into equation 10. The resulting G.F was  $2.190 \pm 0.005$ . In comparison to the theoretical G.F, 2.08, the percent error was 5.0%, which means that the relative relationship between the strain gauges were accurate. In addition when SG 4 elasticity was plotted against SG

5 elasticity, the slope that is the Poisson number was 0.4716, compared to the given Poisson's number, 0.33, it has a 30% error.

In part 2 of this experiment the beam was analyzed using the dynamic method. The oscillation frequency was found by using equation 6, taking into account the mass of the beam, and equation 7, neglecting the mass of the beam. Comparing the two theoretical results of equation 6 and 7, it can be noted that the k constant found is nearly the same. When the mass of the beam was taken into account, the k constant was 464.45 N/m, Leading to a 1.126% error in our experimental results. When the mass of the beam was neglected, the spring constant was 464.79 N/m, leading to a 1.052% error in our results. Comparing the spring constant results from the static method value to the results using the dynamic method, it can be seen that the spring constant found using the static method is slightly closer to the experimental value. However, looking at the equations we would have expected the dynamic method to be more accurate as it takes into account the mass of the beam as well as multiple oscillations of the mass. This discrepancy may have been a result of the method used to find the period of oscillation. In the experiment the trace tool was used to calculate the time difference from peak to peak of an oscillation. However when these values were calculated, they varied slightly from the reported period on the oscilloscope. This error may have been the cause of the slight offset from the theoretical value.

### Conclusion:

When determining the spring constant using the static method, the k constant was 464.79 N/m and when using the dynamic method, k was 469.68 N/m. Young's modulus of aluminum, E, was determined to be  $42.7 \pm 0.4$  GPa, resulting in a 38% error from the expected value. Next, the theoretical and experimental strain values were graphed against each other to display that both had a positive correlation, however there was a large discrepancy between the actual values. The calculated Gauge Factor for SG 3 was calculated by finding the optimal maximum strain and using equation 10. The resulting G.F was  $2.190 \pm 0.005$ . Finally, these graphs were used to find Poisson's ration of aluminum, 0.4716.

### Error Propagation:

All standard deviations were calculated using the following equation:

$$S_M = M \sqrt{\left(\frac{S_X}{X}\right)^2 + \left(\frac{S_Y}{Y}\right)^2 + \left(\frac{S_Z}{Z}\right)^2}$$

Figure 7<sup>2</sup>: General equation for the Standard Deviation for multiplication and division

### Acknowledgements

We would like to thank Professor Farhat Beg, Professor Robert Cattolica and Rich Inman.

### Reference

<sup>1</sup>R. Cattolica, C. Coimbra. "MAE 170 Lecture 7: Measurement of Strain & Force"

<sup>2</sup> R. Cattolica, C. Coimbra. "MAE 170 Lecture 2: Analog A/D conversion, Sampling Rates, Error Analysis and Report Writing"

**Appendix:**

Gauge 1	
Load	V
0	0
1.95219	-0.111
3.925962	-0.094
5.962518	-0.073
7.956891	-0.056

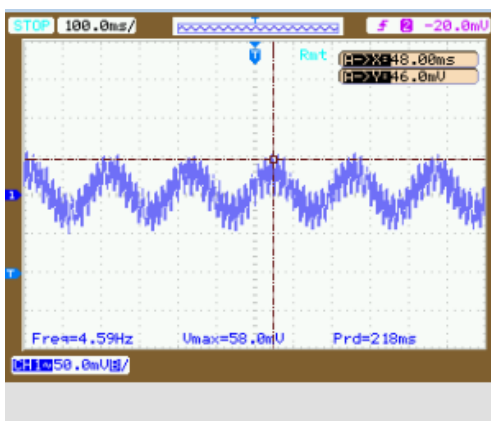
Gauge 2	
Load	V
0	0
1.95219	-0.018
3.925962	-0.005
5.962518	0.017
7.956891	0.056

Gauge 3	
Load	V
0	0
1.95219	-0.028
3.925962	-0.054
5.962518	-0.081
7.956891	-0.107

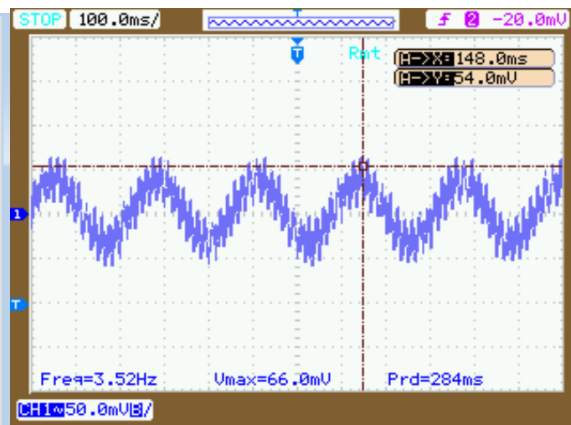
Gauge 5	
Load	V
0	0
1.95219	-0.01
3.925962	-0.024
5.962518	-0.034
7.956891	-0.044

Gauge 4	
Load	V
0	0
1.95219	0.023
3.925962	0.045
5.962518	0.07
7.956891	0.095

**Table 2:** Load and Voltage of Strain Gauges 1-5.

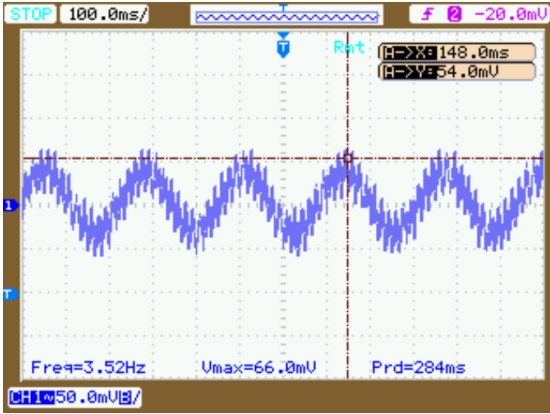


**Figure 8:** Oscillation for 499.2 g

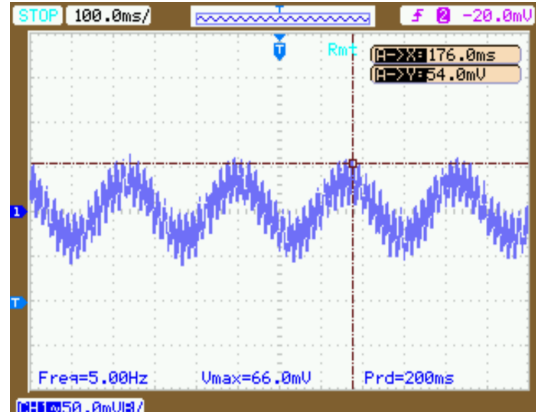


**Figure 9:** Oscillation for 603.5 g

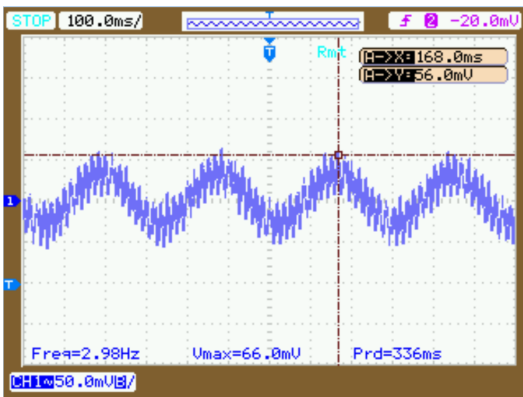




**Figure 10:** Oscillation for 712.9 g



**Figure 11:** Oscillation for 817.3 g



**Figure 12:** Oscillation for 963.2 g